Thermal and Quantum Fluctuations in the Kaon Condensed Phase

Toshitaka Tatsumi and Masatomi Yasuhira *

Department of Physics, Kyoto University

Kyoto 606-8502, Japan

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A new formulation is presented to treat thermal and quantum fluctuations around the kaon condensate on the basis of chiral symmetry; separating the zero-mode from the beginning and using the path-integral method, we can formulate the inclusion of fluctuations in a transparent way. Nucleons as well as kaons are treated in a self-consistent way to the one-loop order. The effects of the Goldstone mode, stemming from the breakdown of $V-$spin symmetry in the condensed phase, are figured out.

A procedure is discussed to renormalize the divergent integrals properly up to the one-loop order. Consequently the thermodynamic potential is derived. It is pointed out that the zero-point fluctuation by nucleons gives a sizable effect, different from the kaonic one.

I. INTRODUCTION

It has been extensively discussed that kaons, the lightest hadrons with strangeness, show characteristic features through the kaon-nucleon interactions once immersed in nuclear medium. The scalar interaction (called the sigma term) and the vector interaction (called the Tomozawa-Weinberg term) are the leading ones, and they bring about a large attraction for the excitation energy of K^- mesons at low momentum, whereas a repulsion for K^+ mesons. There are many proposals to see these consequences experimentally; K^- enhancement in the subthreshold kaon production or the anomaly around the ϕ meson peak in the dilepton production from the relativistic heavy-ion collisions are some of them [1–3].

In 1986 Kaplan and Nelson suggested a possibility of kaon condensation in dense matter by using a chiral Lagrangian [4]. Since then this subject has been studied by many authors [5]. The s-wave attractions between kaons and nucleons remarkably reduce the excitation energy of K^- mesons in neutron matter and thereby kaons should condense when the degeneracy energy of electrons exceeds it. It is well-known that the kaon condensation results in the large softening of equation of state (EOS) and the inducement of new processes due to the presence of the condensate. Since it may occur at relatively low densities, $(3-4)\rho_0$ with ρ_0 being the nuclear density $\simeq 0.16\text{fm}^{-3}$ in neutron star matter, it is believed to have many implications on neutron star physics; the fast cooling mechanism, rapid rotation or the low-mass black hole scenario are some of them [6–8].

We have studied this subject from the point of view of chiral rotation on the chiral manifold G/H with $G = SU(3)_L \times SU(3)_R, H = SU(3)_V$ [9]. We have defined the kaon condensed state by acting a unitary operator $\hat{U}$ on the normal matter $|0\rangle$ (the meson vacuum); $|K\rangle = \hat{U}_K(\langle\theta\rangle)|0\rangle$ with $\hat{U}_K(\langle\theta\rangle) = \exp(i\langle\theta\rangle\hat{F}_4^5)$, where $\hat{F}_4^5$ is the 4-th component of the axial-vector charge of the $SU(3)_L \times SU(3)_R$ algebra and $\langle\theta\rangle$ is the chiral angle. [1] Since the kaon condensed state is a chirally rotated one, the change of the kaon-nucleon dynamics from that in normal matter arises from the symmetry breaking terms in the Hamiltonian. Thus we have succeeded in extracting the essential features of kaon condensation in this formalism.

So far almost all the studies have been done within the mean-field theory, the classical approximation. Recently there begin to appear studies about kaon condensation in protoneutron stars, where thermal effects or roles of the neutrino trapping become important and we must treat it beyond the mean-field level [12]. Low-mass black hole, if exists, may be produced in this era [8]. A numerical simulation based on the general

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*E-mail address: tatsumi@ruby.scphys.kyoto-u.ac.jp;   yasuhira@ruby.scphys.kyoto-u.ac.jp
[1]The notation $\langle\theta\rangle$ implies the thermal average of θ.

relativity has been already done about the delayed collapse of protoneutron stars to the low-mass black hole [11]. However, they used a cold EOS for the kaon condensed phase, while temperature (T) rises to several tens MeV there. Also, in this era, various time-scales become important; collapsing, neutrino-trapping or initial cooling time-scale may be relevant. Hence we cannot assume chemical equilibrium *a priori* for such hot matter since it may be achieved through *weak* interactions. A problem how condensate appears and grows to establish chemical equilibrium is an interesting subject to pursure [10]. Prakash et al. treated the kaon-condensed phase at finite temperature and discussed the properties of protoneutron stars within the meson exchange model, since there is no consistent theory based on chiral symmetry [12].

In a recent paper, Thorsson and Ellis have tried to include quantum or thermal fluctuations within the heavy-baryon chiral perturbation theory (HBCPT) [15], frozing dynamical degrees of freedom of nucleons to the thermodynamic potential [13]. However, they have discussed only the effect of the zero-point fluctuation of kaons at $T = 0$ to the one-loop order. The most interesting result in their paper may be the dispersion relation of kaons in the condensed phase, which is a key ingredient for the discussion of thermal properties of the condensed phase. Moreover, it plays an important role in the discussion of the relaxation process from the nonequilibrium state to the kaon condensed state in equilibrium [10]. They have used the Kaplan-Nelson Lagrangian, where the basic variable is the $SU(3)$ matrix specified by the eight Goldstone fields ϕ_a, $U(\phi_a) = \exp(2iT_a\phi_a)$ with T_a being the generators of $SU(3)$ Lie algebra. They picked up only the degrees of freedom for the charged kaon fields $K^\pm$, namely ϕ_4 and ϕ_5, and separated the classical field (the condensate) and the fluctuation field around it in the standard manner, $K^\pm = \langle K^\pm \rangle + \tilde{K}^\pm$. The resultant dispersion relation for kaons has a very complicated form due to the interaction between the condensate and fluctuations. Accordingly other thermodynamic quantities also become too complicated to be tractable. Thus there is no consistent finite temperature calculation for the chiral case.

In ref. [14] we have presented a new formalism to treat the kaon condensation at finite temperature within HBCPT. We, in this paper, extend the formalism to take into account the dynamical degrees of nucleons as well as kaons [16]. We introduced fluctuations around the condensate by acting the chiral transformation on the chiral manifold successively; after generating fluctuations around the meson vacuum by a chiral transformation (η) we transform this state further by another chiral rotation (ζ), which should be the same as the classical one (see Fig.1).
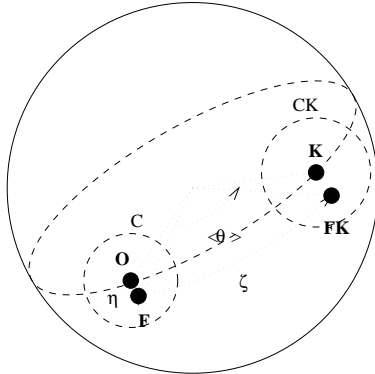

FIG. 1. Chiral rotations on the $SU(3)$ manifold. Circles C, CK indicate the fluctuation "areas" around the vacuum (O) and kaon condensed state (K), respectively.

Since the constant condensate can be considered as a kind of zero-mode, this procedure can be regarded as a separation of the zero-mode from the original matrix $U(\phi_a)$. Similar procedure has been applied in the different context to see the finite-volume effects of the chiral symmetry restoration [19]. Then we can obtain the dispersion relation for the kaon excitation around the condensate. We have seen that the dispersion relation shows the similar feature to the one by Thorsson and Ellis, whereas its form is very different. We have also proposed an approximation which makes the expressions of the thermodynamic quantities considerably simple. We have checked numerically that this approximation may work very well.

In this paper we treat the dynamical degrees of freedom of kaons and nucleons consistently; since nucleons interact with kaons, their single-particle energies are modified not only by the condensate but also by the fluctuations. For this purpose we use the Hartree approximation for the four-point vertices among kaons and nucleons; the nucleon Green function contains kaon loops and the kaon Green function does nucleon loops. Therefore we sum up the infinite series of bubble diagrams composed of one-loop diagrams by kaons

and nucleons in a self-consistent way. We shall see that our formalism makes the analysis of the structure of kaon-nucleon (KN) dynamics transparent and clarifies physics included there.

Generally these loop diagrams include divergences, which should exist even at $T = 0$. We discuss how finite contributions can be extracted in our formalism. Analyzing the structure of the self-energy terms, we show these divergences are properly renormalized to the one-loop order by introducing relevant counterterms corresponding to the nucleon and kaon masses, and the KN and KK interaction vertices. Then no more counterterms are needed to renormalize the thermodynamic potential. Effects of the zero-point (quantum) fluctuation by kaons has been shown to be small [13,14]. We shall see that the zero-point fluctuation by nucleons has a sizable effect on the ground-state property of the condensed phase.

In II we present our formalism to treat fluctuations around the condensate by way of the path integral approach. In III we develop our approach and perform the one-loop calculation. There we get the dispersion relation for kaons which is a key object to discuss the properties of the kaon condensed phase at finite temperature. We discuss in IV how the thermodynamic potential is renormalized by analyzing the self-energy terms. We also give some numerical results about the zero-point fluctuation by nucleons. V is devoted to some discussions and concluding remarks. In Appendix A we discuss the separation of zero-modes, one of which is the would-be condensate. Some results within the mean-field approximation are summarized in Appendix B. In Appendix C we give some properties of the kaon Green function and expressions of the loop contributions.

II. PATH INTEGRAL FORMULATION

A. Partition function

We start with the Kaplan-Nelson Lagrangian as a chiral Lagrangian [24],

$$\mathcal{L}_{chiral} = \mathcal{L}_0 + \mathcal{L}_{SB}, \tag{2.1}$$

where $\mathcal{L}_0$ is the symmetric part,

$$\mathcal{L}_0 = \frac{f^2}{4}\text{tr}(\partial_\mu U^\dagger \partial^\mu U) + i\text{tr}\{\bar{B}(\slashed{D} - m_B)B\} + D\text{tr}(\bar{B}\gamma_\mu\gamma_5\{A^\mu, B\}) + F\text{tr}(\bar{B}\gamma_\mu\gamma_5[A^\mu, B]). \tag{2.2}$$

and $\mathcal{L}_{SB}$ the symmetry breaking part;

$$\mathcal{L}_{SB} = v\text{tr}\hat{m}_q(U + U^\dagger - 2)$$
$$+ a_1\text{tr}\bar{B}(\xi\hat{m}_q\xi + h.c.)B + a_2\text{tr}\bar{B}B(\xi\hat{m}_q\xi + h.c.) + a_3\{\text{tr}\bar{B}B\}\text{tr}(\hat{m}_q U + h.c.) \tag{2.3}$$

with the quark mass-matrix, $\hat{m}_q \simeq \text{diag}(0, 0, m_s)$. The coefficient v implies the vacuum expectation value of the quark bilinear, $-v = \langle 0|\bar{q}q|0\rangle$, and the coefficients a_i measure the strength of the explicitly symmetry breaking: the kaon mass is given as $m_K^2 \simeq vm_s/f^2$ and the KN sigma terms as $\Sigma_{Kp} = -m_s(a_1 + a_2 + 2a_3)$ and $\Sigma_{Kn} = -m_s(a_2 + 2a_3)$. U is the $SU(3)$ matrix parametrized by the eight Goldstone fields ϕ_a,

$$U = \exp[2iT_a\phi_a/f] \in G/H \tag{2.4}$$

and V_μ, A_μ are the vector and axial-vector fields constructed by U,

$$V_\mu \equiv 1/2(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger)$$
$$A_\mu \equiv i/2(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger), \tag{2.5}$$

with $U = \xi^2$. The baryon octet B can be represented by the $SU(3)$ matrix,

$$B = \begin{bmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & \frac{-2\Lambda^0}{\sqrt{6}} \end{bmatrix}.$$

Then the covariant derivative D_μ is defined by the vector field V_μ;

$$D_\mu B \equiv \partial_\mu B + [V_\mu, B]. \tag{2.6}$$

The transformation properties of the fields U and B under G are found in, e.g., ref. [24].

To treat thermal and quantum fluctuations consistently we use here the path-integral formulation. It is well-known that chemical potentials μ_a, which should be taken into account to ensure various conservation laws in the ground state, can be introduced by "gauging" the Lagrangian with artificial "gauge fields" $A_\mu \equiv T_a A^\mu$ with $A_a^\mu = (\mu_a, \mathbf{0})$. Here we must take into account conservation of two quantities: electromagnetic charge and baryon number. Accordingly we consider two chemical potentials, the charge chemical potential and baryon chemical potential. For the charge chemical potential μ_K, which we shall see to be identified as the kaon chemical potential, the "covariant derivative" $\mathcal{D}_\mu U$ for the Goldstone field then reads

$$\mathcal{D}_{\mu=0} U = \partial U / \partial t + i\mu_K [T_{em}, U], \quad \mathcal{D}_\mu U = \partial_\mu U \quad \text{for} \quad \text{others,} \tag{2.7}$$

with the charge operator, $T_{em} \equiv T_3 + 1/\sqrt{3} T_8 = \text{diag}(2/3, -1/3, -1/3)$. For baryon field B, we must introduce another Abelian gauge field to ensure the conservation of baryon number, $B_\mu = (\mu_n, \mathbf{0})$ with the baryon chemical potential μ_n, which we shall see to be the neutron chemical potential. Then the "covariant derivative" $\nabla_{\mu=0} B$ for the baryon field reads

$$\nabla_{\mu=0} B = \partial B / \partial t - i\mu_n B + i\mu_K [T_{em}, B] + [V_0, B], \quad \nabla_\mu B = D_\mu B \quad \text{for} \quad \text{others.} \tag{2.8}$$

Therefore the partition function in the imaginary-time formalism can be represented as follows ($\tau = it, \beta = 1/T$);

$$Z_{chiral} = N \int [dU][dB][d\bar{B}] \exp[S'_{chiral}] \tag{2.9}$$

$$S'_{chiral} = \int_0^\beta d\tau \int d^3x \mathcal{L}_{chiral} (\mathcal{D}_\mu U, U, \nabla_\mu B, B) = \int_0^\beta d\tau \int d^3x [\mathcal{L}_{chiral} + \delta\mathcal{L}], \tag{2.10}$$

where the "time-derivative" should be read as the derivative with respect to imaginary time. Here we can see that there appears an additional term $\delta\mathcal{L}$ besides the original chiral Lagrangian $\mathcal{L}_{chiral}$, which is in general *non-invariant* under the chiral transformation. For the Lagrangian (2.1), we find

$$\delta\mathcal{L} = -\frac{f^2 \mu_K}{4} \text{tr}\{[T_{em}, U]\frac{\partial U^\dagger}{\partial \tau} + \frac{\partial U}{\partial \tau}[T_{em}, U^\dagger]\} - \frac{\mu_K}{2} \text{tr}\{B^\dagger [(\xi^\dagger [T_{em}, \xi] + \xi[T_{em}, \xi^\dagger]), B]\}$$

$$- \frac{f^2 \mu_K^2}{4} \text{tr}\{[T_{em}, U][T_{em}, U^\dagger]\} + \mu_n \text{tr}\{B^\dagger B\} - \mu_K \text{tr}\{B^\dagger [T_{em}, B]\}. \tag{2.11}$$

Here the first term corresponds to the mesonic charge, and the last two terms indicate nothing else but the baryon number and the electromagnetic charge of baryons.

B. Transformation of the coordinates

We show our formulation to include thermal and quantum fluctuations in the chiral Lagrangian. Before that let us recall the tree case. We have described the kaon condensed state $|K\rangle$, which is specified by the chiral angle $\langle\theta\rangle$, the order parameter of kaon condensation, as a chiral-rotated state [9];

$$|K\rangle = \hat{U}_K(\langle\theta\rangle)|0\rangle \tag{2.12}$$

with the operator $\hat{U}_K(\langle\theta\rangle) = \exp(i\hat{F}_4^5 \langle\theta\rangle)$, where $\hat{F}_4^5$ is the 4-th element of the axial-vector charge in $SU(3)_L \times SU(3)_R$ algebra and $|0\rangle$ represents the normal nuclear matter. We can also describe this point on the $SU(3)_L \times SU(3)_R / SU(3)_V \simeq SU(3)$ manifold which is coordinated by the Goldstone fields, $U(\phi_a) = \exp(2iT_a \phi_a / f) \in SU(3)$ and the meson vacuum $|0\rangle$ corresponds to the fiducial point on the manifold by the spontaneous symmetry breaking. Then kaon condensed state Eq. (2.12) corresponds to the $SU(3)$ matrix $U_K = \exp[2i\{T_4 \langle\phi_4\rangle + T_5 \langle\phi_5\rangle\}/f]$, and can be represented as a chiral rotated one from the vacuum on this manifold,

$$U_K(\langle\theta\rangle) = \zeta U_V \zeta = \zeta^2 \tag{2.13}$$

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where $U_V = U(\phi_a = 0) = 1$ corresponds to the meson vacuum and ζ is the operator, $\zeta = \exp(i\langle M\rangle/\sqrt{2}f)$ with the constant matrix

$$M = \begin{bmatrix} 0 & 0 & K^+ \\ 0 & 0 & 0 \\ K^- & 0 & 0 \end{bmatrix}, \qquad K^\pm = (\phi_4 \pm i\phi_5)/\sqrt{2} \qquad \text{and} \qquad \theta^2 \equiv 2K^+K^-/f^2.$$

Thus we have seen that the classical kaon field $\langle K^\pm \rangle$ induces the chiral rotation on the manifold [9].
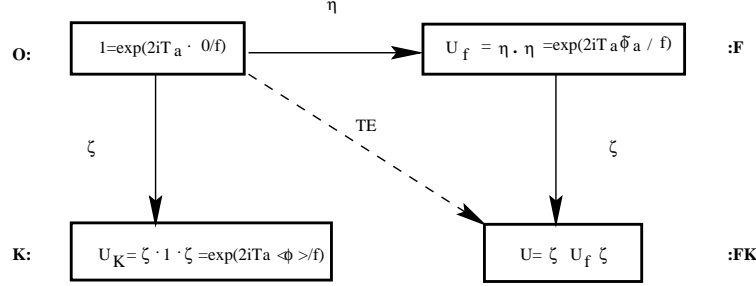

FIG. 2. Diagram showing the relation among the transformations on the chiral manifold (c.f. Fig. 1). The symbol TE indicates the one by Thorsson and Ellis [13].

When we introduce the quantum or thermal fluctuation, there may be several ways (Figs.1,2). Thorsson and Ellis introduced the kaon fluctuation field $\tilde{K}^\pm$ as a deviation from the classical kaon field in the standard manner, [2]

$$K^\pm = \langle K^\pm \rangle + \tilde{K}^\pm. \tag{2.14}$$

Here we introduce the fluctuation fields by developing the idea of chiral rotation mentioned above. First, we notice that any fluctuation of the Goldstone fields can be introduced by a chiral rotation U_f from the vacuum $U_V = 1$,

$$\eta: \qquad U_f = \eta U_V \eta = \eta^2 \tag{2.15}$$

with $\eta = \exp(iT_a\phi_a/f)$. The subsequent chiral rotation by ζ transforms U_f into the form,

$$\zeta: \qquad U(\langle\theta\rangle, \phi_a) = \zeta U_f(\phi_a)\zeta. \tag{2.16}$$

We can easily see that in the limit $\phi_a \to 0$, which corresponds to the previous classical approximation, the matrix U is reduced to U_K (2.13). Thus we have introduced the fluctuation fields by the two-step procedure. [3] It is to be noted that the constant component in ϕ_a is redundant since it can be always absorbed into ζ by redefinition. It may be also worth noting that we can easily take into account fluctuations of *any* meson field in the kaon condensed state by the form of Eq. (2.16). [4] Accordingly, ξ operator, which is defined by $U = \xi^2$, can be obtained by solving the subsidiary equation,

$$\xi = \zeta\eta u^\dagger = u\eta\zeta, \tag{2.17}$$

where the matrix u is defined by the second equality in Eq.(2.17). It depends on $\phi_a, \langle\theta\rangle$ in a complicated nonlinear way, and thereby it is very difficult to find u for the general form of U_f [24]. A systematic way to find the form of u may be the perturbative method: expanding the matrices η, u with respect to the

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[2]Hereafter, we omit the tilde on the field for the sake of simplicity.

[3]The similar decomposition was also used in the context of the loop expansion within chiral perturbation theory [16] or the bound-state approach to the Skyrme model [17].

[4]Alternatively, we can regard this procedure as a separation of the "zero-mode" from the full $SU(3)$ matrix $U(\phi_a)$ (see Appendix A). Using Eq. (A17) we can see that the value of the thermal average $\langle\theta\rangle$ should be determined by imposing the extremum condition for the resultant thermodynamic potential at the end.

fluctuation field ϕ_a, we can solve Eq. (2.17) order by order. Hence we can find the relevant form of u up to any order of ϕ_a, depending on the problem or the approximation we are interested in. On the other hand, if we restrict ourselves to the fluctuations of kaon sector s.t.

$$U_f = \exp(i\sqrt{2}M/f), \tag{2.18}$$

then we can find u in the closed form,

$$u = \text{diag}(\kappa^*/|\kappa|, 1, \kappa/|\kappa|), \tag{2.19}$$

with

$$\kappa = \cos(\langle\theta\rangle/2)\cos(\theta/2) - \frac{K^+\langle K^-\rangle}{|\langle K\rangle||K|}\sin(\langle\theta\rangle/2)\sin(\theta/2). \tag{2.20}$$

Here the following relation is to be noted,

$$u^\dagger T_{em} u = T_{em}. \tag{2.21}$$

Defining a new baryon field B' by the use of the matrix u,

$$B' = u^\dagger B u, \tag{2.22}$$

we can see that the chiral-invariant pieces of the Lagrangian are not changed and only the symmetry breaking pieces play essential roles,

$$\mathcal{L}_{chiral}(U_K, B) = \mathcal{L}_0(U_K, B) + \mathcal{L}_{SB}(U_K, B) \longrightarrow \mathcal{L}_0(U_f, B') + \mathcal{L}_{SB}(\zeta U_f \zeta, u B' u^\dagger)$$
$$\delta\mathcal{L}(U_K, B) \longrightarrow \delta\mathcal{L}(\zeta U_f \zeta, u B' u^\dagger). \tag{2.23}$$

Thus all the dynamics of kaons and baryons in the condensed phase are completely prescribed by the non-invariant terms under chiral transformation. It is to be noted that this feature is quite similar to the one within the classical approximation at $T = 0$ [9].

Using Eqs. (2.19), (2.22), we find

$$\begin{aligned}\delta\mathcal{L}(\zeta U_f \zeta, u B' u^\dagger) =\ &-(\cos(\langle\theta\rangle - 1)\mu_K \text{tr}\{B'^\dagger[V_3, B']\} + f^2\mu_K^2/2\sin^2\langle\theta\rangle \\ &- \mu_K \cos\langle\theta\rangle(K^+\dot{K}^- - \dot{K}^+ K^-) + \mu_K \cos\langle\theta\rangle K^+ K^-/f^2 \text{tr}\{B'^\dagger[V_3, B']\} \\ &+ \cos^2\langle\theta\rangle\mu_K^2 K^+ K^- - \mu_K^2/4\sin^2\langle\theta\rangle(\exp(i\gamma)K^- + \exp(-i\gamma)K^+)^2 \\ &+ \mu_n \text{tr}\{B'^\dagger B'\} - \mu_K \text{tr}\{B'^\dagger[T_{em}, B']\}..., \end{aligned} \tag{2.24}$$

with the V-spin operator, $V_3 = 1/2(T_3 + \sqrt{3}T_8)$, and $\dot{K}^\pm = \partial K^\pm/\partial\tau$, where the phase γ is defined by $\exp(\pm i\gamma) = \langle K^\pm\rangle/|\langle K\rangle|$, which can be absorbed into the fluctuation field by redefining it, $\exp(\mp i\gamma)K^\pm \to K^\pm$. In other words the phase of the condensate is arbitrary, reflecting V_3 symmetry in the chiral Lagrangian, and we can choose $\gamma = 0$ without loss of generality. As a consequence V_3 symmetry is spontaneously broken in the condensed phase, and thereby we shall see the Goldstone mode associated with this symmetry (see IV). [5]

If $\mathcal{L}_{SB}$ originates from the quark mass term , $\mathcal{L}_{mass} = -\bar{q}\hat{m}_q q$, we can further explore the transformation property of $\mathcal{L}_{SB}$ in a model-independent way: first we can see

$$\mathcal{L}_{mass} \to \mathcal{L}_{mass} - i\sin\theta[\hat{F}_4^5, \mathcal{L}_{mass}] + (\cos\theta - 1)\Sigma_K \tag{2.25}$$

with the sigma term $\Sigma_K = [\hat{F}_4^5, [\hat{F}_4^5, \mathcal{L}_{mass}]]$, under the transformation by η in Eq. (2.15). Similarly, $\mathcal{L}_{mass}$ transforms like (2.25) with $\langle\theta\rangle$ instead of θ under the chiral rotation ζ in Eq. (2.16). Hence $\mathcal{L}_{mass}$ eventually transforms under the successive chiral transformations (2.15) and (2.16) like

$$\mathcal{L}_{mass} \to \mathcal{L}_{mass} - i\left(\sin(\langle\theta\rangle + \theta) - \sin\theta\right)[\hat{F}_4^5, \mathcal{L}_{mass}] + \left((\cos(\langle\theta\rangle + \theta) - \cos\theta\right)\Sigma_K, \tag{2.26}$$

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[5]We met the similar situation in the context of pion condensation [21].

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where we have used the property $[\hat{F}_4^5, [\hat{F}_4^5, [\hat{F}_4^5, \mathcal{L}_{mass}]]] = [\hat{F}_4^5, \mathcal{L}_{mass}]$. The second term is a parity-odd operator and irrelevant due to the fact, $\langle \bar{q}\gamma_5 q \rangle = 0$. The term proportional to $\sin\langle\theta\rangle$ included in the last term is also irrelevant in the following calculation bacause of the thermal-equilibrium condition (c.f. Eq. (A17)). Therefore there is essentially left the term,

$$\mathcal{L}_{mass} \to \mathcal{L}_{mass} + (\cos\langle\theta\rangle - 1)\cos\theta\Sigma_K. \tag{2.27}$$

Accordingly, in terms of the effective Lagrangian $\mathcal{L}_{SB}$, it should transform like

$$\mathcal{L}_{SB}(\zeta U_f \zeta, uB'u^\dagger) = \mathcal{L}_{SB}(U_f, B') + (\cos\langle\theta\rangle - 1)\cos\theta\Sigma_K(1, B') \tag{2.28}$$

besides the irrelevant terms, where $\Sigma_K(1, B')$ has a standard form,

$$\Sigma_K(1, B') = f^2 m_K^2 - \Sigma_{Kp}\bar{p}'p' - \Sigma_{Kn}\bar{n}'n', \tag{2.29}$$

for nuclear matter. [6] In Eq. (2.29) we have used the following notations: $f^2 m_K^2 = v[m_u + m_s], \Sigma_{Kp(n)} = \langle p(n)|\Sigma_K|p(n)\rangle$. This consequence is rather general and, of course, consistent with that in the Kaplan-Nelson Lagrangian (2.3).

For the integration measure ($N = 3$), it is given as

$$[dU] = M(\phi) \prod_{i=1}^{N^2-1} [d\phi_i], \tag{2.30}$$

where $M(\phi)$ is the Lee-Yang term [22], $M(\phi) = \exp\left[\frac{1}{2}\delta^{(4)}(0)\int_0^\beta d\tau \int d^3x \ln g(\phi)\right]$ with the Cartan-Killing metric on the $SU(N)$ manifold from the invariant line element, $ds^2 = g_{ik}(\phi)d\phi_i d\phi_k = 1/2\text{tr}[dU dU^\dagger]$. It is to be noted that the Lee-Yang term is important to make the integration measure chiral invariant. [7] The measure (2.30) is invariant under the transformation (2.16) by ζ,

$$[dU] = [d(\zeta U_f \zeta)] = [dU_f], \tag{2.31}$$

because G is the isometry, $\text{tr}[\zeta dU_f \zeta \zeta^\dagger dU_f^\dagger \zeta^\dagger] = \text{tr}[dU_f dU_f^\dagger]$, so that the geometry around the condensed point is the same as in the vicinity of the vacuum. Since we are concerned with the fluctuations, the measure can be approximated by the one with flat curvature [19], $[dU_f] \simeq \prod_{i=1}^{N^2-1}[d\phi_i]$. In this case we are not worried by the Lee-Yang term because of $g = 1 + O(\phi_i^2/f^2)$. Thus we find

$$Z_{chiral} \simeq \int \prod_{i=1}^{N^2-1} [d\phi_i][dB'][d\bar{B}'] \exp[S_{chiral}^{eff}(\zeta, U_f, B')]. \tag{2.32}$$

III. THERMODYNAMIC POTENTIAL

A. Partition function for a chiral Lagrangian

Using the formulation given in II we evaluate the partition function in the kaon condensed state. [8] In the following calculation we only consider the one-loop diagrams, and thereby we retain only the quadratic

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[6] We, hereafter, omit the prime on the nucleon fields for simplicity.

[7] In ref. [13], their treatment of the Lee-Yang term is obscure: $\gamma^{1/2}$ in Eq (13) in ref. [13], which corresponds to g in our notation, should read $\exp\left[\frac{1}{2}\delta^{(4)}(0)\int_0^\beta d\tau \int d^3x \ln \gamma\right]$.

[8] We briefly discuss the case of the classical approximation in Appendix B, where we can see that our previous results given in ref. [9] are recovered in a model-independent way.

terms with respect to the kaon field. Moreover, we are concerned here with only charged-kaon fluctuations and nuclear matter without hyperons. [9] Then we can write

$$Z_{chiral} = N \int [d\phi_4][d\phi_5][d\psi^\dagger][d\psi] \exp[S_{chiral}^{eff}] \tag{3.1}$$

with the spinor $\psi = \begin{pmatrix} p \\ n \end{pmatrix}$ by the use of Kaplan-Nelson Lagrangian (2.1).

$$S_{chiral}^{eff} = S_c + S_K + S_N + S_{int}, \tag{3.2}$$

where

$$S_c = \beta V \left[\frac{1}{2}\mu_K^2 f^2 \sin^2\langle\theta\rangle - f^2 m_K^2 (1 - \cos\langle\theta\rangle), \right] \tag{3.3}$$

$$S_K = \int_0^\beta d\tau \int d^3x \left[\partial_\mu K^+ \partial_\mu K^- - \cos\langle\theta\rangle(m_K^2 - \mu_K^2 \cos\langle\theta\rangle)K^+ K^- - \mu_K \cos\langle\theta\rangle K^+ \overset{\leftrightarrow}{\partial_\tau} K^- \right.$$

$$\left. - \frac{\mu_K^2}{4} \sin^2\langle\theta\rangle(K^+ + K^-)^2 + O(|K|^4) \right], \tag{3.4}$$

with the shorthand notation $\partial_\mu K^+ \partial_\mu K^- = -\partial_\tau K^+ \partial_\tau K^- - \boldsymbol{\nabla}K^+ \boldsymbol{\nabla}K^-$,

$$S_N = \int_0^\beta d\tau \int d^3x \left[\bar{\psi}(-\gamma^0 \frac{\partial}{\partial\tau} + i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} - M + \mu_N\gamma^0 \right.$$

$$\left. + \mu_K(1 - \cos\langle\theta\rangle)\gamma^0 \frac{3 + \tau_3}{4} + (1 - \cos\langle\theta\rangle)\Sigma_{KN})\psi \right], \tag{3.5}$$

and S_{int} is the interaction term, which mainly consists of the two terms, the sigma-term and the Tomozawa-Weinberg (TW) term,

$$S_{int} = \int_0^\beta d\tau \int d^3x \left[\frac{1}{f^2} \cos\langle\theta\rangle \bar{\psi}\Sigma_{KN}\psi K^- K^+ \right.$$

$$\left. - \frac{1}{f^2}\bar{\psi}\gamma_0 \frac{3 + \tau_3}{8}\psi \left\{ K^+ \overset{\leftrightarrow}{\partial_\tau} K^- - 2\mu_K \cos\langle\theta\rangle K^+ K^- \right\} + O(|K|^4) \right] \tag{3.6}$$

with the KN sigma term Σ_{KN}. It is to be noted that the sigma-term Σ_{KN} and nucleon chemical potential μ_N are 2×2 diagonal matrices in the isospin space: $\Sigma_{KN} = \begin{pmatrix} \Sigma_{Kp} & 0 \\ 0 & \Sigma_{Kn} \end{pmatrix}$ and $\mu_N = \begin{pmatrix} \mu_p & 0 \\ 0 & \mu_n \end{pmatrix}$ with $\mu_p \equiv \mu_n - \mu_K$, respectively.

Since the effective action S_{chiral}^{eff} is already bilinear with respect to the nucleon field, we can easily integrate out the nucleon degree of freedom in Eq. (3.1),

$$Z_{chiral} = N \int [d\phi_4][d\phi_5] \exp[S_K'] \mathrm{Det} G = N \int [d\phi_4][d\phi_5] \exp[S_K' + \mathrm{Tr}\ln G]. \tag{3.7}$$

with $\tilde{S}_K' = S_c + S_K$ and the Dirac operator G. The capital letter for determinant or trace is used for the Dirac operator G of infinite dimension over space-imaginary-time. The Dirac operator G is composed of two parts, $G = G_0 + \delta G$, with the "free" part

$$G_0 = -\frac{\partial}{\partial\tau} + i\gamma_0\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} - M\gamma_0 + \mu_N + \mu_K(1 - \cos\langle\theta\rangle)\frac{3 + \tau_3}{4} + (1 - \cos\langle\theta\rangle)\gamma_0\Sigma_{KN}, \tag{3.8}$$

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[9]An extension to include hyperons may be straightforward, but note that there is still large ambiguity about their coupling strength with hadrons, e.g. see ref. [12].

and the interaction part

$$\delta G = \frac{\Sigma_{KN}}{f^2}\cos\langle\theta\rangle\gamma_0 K^- K^+ - \frac{1}{f^2}\frac{3+\tau_3}{8}\left\{K^+ \overset{\leftrightarrow}{\partial_\tau} K^- - 2\mu_K\cos\langle\theta\rangle K^+ K^-\right\} + O(|K|^4). \tag{3.9}$$

A naive treatment may proceed with treating G_0 as the zeroth-order operator and expanding $\mathrm{Tr}\ln G$ in terms of δG,

$$\mathrm{Tr}\ln G = \mathrm{Tr}\ln G_0 + \mathrm{Tr}[G_0^{-1}\delta G] + O(|K|^4). \tag{3.10}$$

The first term gives a one-loop contribution of nucleons to the partition function, and the second term to the kaon self-energy term. The latter, together with S_K, gives a one-loop contributions of kaons to the partition function. However, this procedure is not adequate for treating the KN interactions in a consistent way; a simple analysis of the loop counting shows that the kaon one-loop contribution includes an infinite series of nucleon loops, while the nucleon one-loop contribution does not include any kaon loop. In the following we use the Hartree approximation to take into account all the bubble diagrams by both nucleons and kaons non-perturbatively, while the exchange diagrams are discarded. Generally speaking, the exchange diagrams are suppressed in the high-density or high-temperature limit, compared with direct diagrams (see Fig.3 for the three loop case) [27,28].
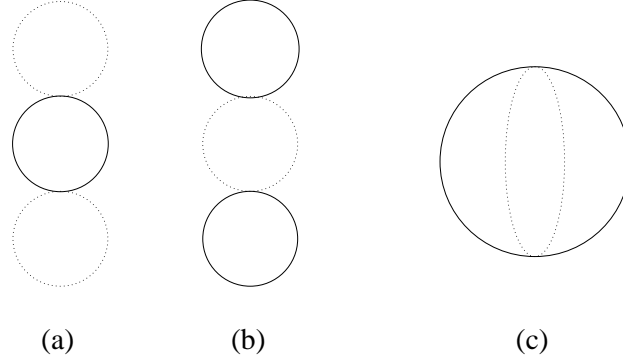

(a)   (b)   (c)

FIG. 3. Three-loop diagrams. (a) and (b) are the direct(bubble) diagrams, which are taken into account by the Hartree approximation. (c) is the exchange diagram, which should be suppressed by the power of density or temperature in comparison with (a) or (b).

We define a new operator G_H, $G_H = G_0 + \langle\delta G\rangle$ by taking the thermal avarage of δG for kaons,

$$G_H = -\frac{\partial}{\partial\tau} + i\gamma_0\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + \mu_N + \frac{3+\tau_3}{8}(n_K^2 + 2\mu_K(1-\cos\langle\theta\rangle))) - M^*\gamma^0 \tag{3.11}$$

with the effective mass of the nucleon, $M^* = \begin{pmatrix} M_p^* & 0 \\ 0 & M_n^* \end{pmatrix}$ with

$$M_i^* = M - \Sigma_{Ki}/f^2\cos\langle\theta\rangle n_K^1 - \Sigma_{Ki}(1-\cos\langle\theta\rangle). \tag{3.12}$$

The quantities n_K^1, n_K^0 mean the thermal kaon-loop contributions for the self-energy of nucleons and they can be represented in terms of the thermal Green function of kaons (see Appendix C) [10] ; $n_K^1 \equiv \langle K^+ K^-\rangle$ and $n_K^2 \equiv -n_K^0 + 2\mu_K\cos\langle\theta\rangle n_K^1$ with $n_K^0 \equiv \langle K^+ \overset{\leftrightarrow}{\partial_\tau} K^-\rangle$.

Using G_H we can rewrite Eq. (3.10) as

$$\mathrm{Tr}\ln G = \mathrm{Tr}\ln G_H + \mathrm{Tr}[G_H^{-1}\delta G] - \mathrm{Tr}[G_H^{-1}\langle\delta G\rangle] + O(|K|^4). \tag{3.13}$$

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[10]The quantity n_K^1 includes the divergence and should be properly renormalized. We shall meet other quantities that also include some divergences in this section. We leave the renormalization problem in IV and hereafter treat them schematically in this section.

Substituting Eq. (3.13) into Eq. (3.7), we find

$$Z_{chiral} = N \int [d\phi_4][d\phi_5] \exp[\tilde{S}_K] \mathrm{Det} G_H, \tag{3.14}$$

with $\tilde{S}_K = S_c + S_K^{eff} - \mathrm{Tr}[G_H^{-1}\langle \delta G \rangle] + O(|K|^4)$. Thus we construct an effective action for the kaon fluctuation field,

$$S_K^{eff} = \int_0^\beta d\tau \int d^3x \quad [-\partial_\mu K^+ \partial_\mu K^- + \cos\langle\theta\rangle(\mu_K^2 \cos\langle\theta\rangle - m_K^2 + \sigma + 2b\mu_K)K^+ K^-$$

$$- (\mu_K \cos\langle\theta\rangle + b)K^+ \overleftrightarrow{\partial}_\tau K^- - \frac{\mu_K^2}{4}\sin^2\langle\theta\rangle(K^+ + K^-)^2 + O(|K|^4)], \tag{3.15}$$

where $\sigma \equiv f^{-2}\mathrm{Tr}[G_H^{-1}\gamma_0 \Sigma_{KN}]$ and $b \equiv f^{-2}\mathrm{Tr}[G_H^{-1}(3+\tau_3)/8] = (4f^2)^{-1}(\rho_n + 2\rho_p)$ with nucleon number densities $\rho_i (i=n,p)$. It may be worth noting that the self-energy term σ contains some divergences (see IV), and is reduced to $\sigma_{tree} \simeq f^{-2}(\Sigma_{Kp}\rho_p + \Sigma_{Kn}\rho_n)$ in the static limit of nucleons [14].

Expanding the fluctuation field according to

$$\phi_{4,5} = \sqrt{\frac{\beta}{V}} \sum_{n,\mathbf{p}\neq 0} e^{i(\mathbf{p}\cdot\mathbf{r}+\omega_n\tau)}\phi_{4,5}(n,\mathbf{p}) \tag{3.16}$$

with the Matsubara frequency $\omega_n = 2\pi nT$, we find

$$Z_K^{eff} = \int \prod_{n,\mathbf{p}\neq 0} [d\phi_4(n,\mathbf{p})][d\phi_5(n,\mathbf{p})]e^{S_K^{eff}}. \tag{3.17}$$

It is to be noted that the zero mode ($n=0, \mathbf{p}=0$) should be removed to avoid the double counting (see Eq. (2.16)). Here the effective action reads

$$S_K^{eff} = -\frac{1}{2}\sum_{n,\mathbf{p}\neq 0}(\phi_4(-n,-\mathbf{p}),\phi_5(-n,-\mathbf{p}))D^{eff}\begin{pmatrix}\phi_4(n,\mathbf{p})\\\phi_5(n,\mathbf{p})\end{pmatrix} \tag{3.18}$$

with the inverse thermal Green function,

$$D^{eff} = \beta^2 \begin{pmatrix} \omega_n^2 + (\tilde{\omega}_- - \tilde{\mu}_K)(\tilde{\omega}_+ + \tilde{\mu}_K) + \mu_K^2\sin^2\langle\theta\rangle & 2(\tilde{\mu}_K + b)\omega_n \\ -2(\tilde{\mu}_K + b)\omega_n & \omega_n^2 + (\tilde{\omega}_- - \tilde{\mu}_K)(\tilde{\omega}_+ + \tilde{\mu}_K) \end{pmatrix}, \tag{3.19}$$

where

$$\tilde{\omega}_\pm = \pm b + (p^2 + C^2)^{1/2} \tag{3.20}$$

with $C^2 = \tilde{m}_K^{*2} + b^2$ and $\tilde{\mu}_K = \mu_K \cos\langle\theta\rangle$ with the effective mass, $\tilde{m}_K^{*2} = \cos\langle\theta\rangle[m_K^2 - \sigma]$. It is to be noted that the effects of the condensate come in through the modifications of the chemical potential, $\mu_K \to \mu_K \cos\langle\theta\rangle$, and the effective mass, $m_K^{*2} = [m_K^2 - \sigma] \to \cos\langle\theta\rangle m_K^{*2}$, except the additional term, $\mu_K^2 \sin^2\langle\theta\rangle$. Then, the partition function can be simply represented as

$$\ln Z_{chiral} = S_c - \mathrm{Tr}[G_H^{-1}\langle\delta G\rangle] - 1/2\mathrm{Tr}'\ln D^{eff} + \mathrm{Tr}\ln G_H, \tag{3.21}$$

where the symbol Tr' means that the zero-mode should be removed.

B. Dispersion relations for kaonic modes

The excitation energy of the kaonic modes are given by the solutions $\omega \equiv i\omega_n$, which satisfy $|D^{eff}| \equiv \det(D^{eff}) = 0$ (see Appendix B):

$$\omega^4 - 2(c_1 + c_3 + 2c_2^2)\omega^2 + 2c_1c_3 + c_1^2 = 0, \tag{3.22}$$

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where $c_1 = (\tilde{\omega}_- - \tilde{\mu}_K)(\tilde{\omega}_+ + \tilde{\mu}_K)$, $c_2 = \tilde{\mu}_K + b$ and $c_3 = 1/2\mu_K^2 \sin^2\langle\theta\rangle$. Then we have two solutions which correspond to the $K^\pm$ modes,

$$E_\pm^2 = (c_1 + c_3 + 2c_2^2) \pm \sqrt{(c_3 + 2c_2^2)^2 + 4c_1 c_2^2}. \tag{3.23}$$

In the limit $\langle\theta\rangle = 0$, $c_1 \to (\omega_- - \mu_K)(\omega_+ + \mu_K)$, $c_2 \to \mu_K + b$ and $c_3 \to 0$, with the dispersion relation in the normal phase, $\omega_\pm(\mathbf{p}) = \pm b + (p^2 + m_K^{*2} + b^2)^{1/2}$. Then $E_\pm^2 \to (\omega_\pm \pm \mu_K)^2$, which recover the previous results as they should do.

In the condensed phase ($\langle\theta\rangle \neq 0$) the mode corresponding to E_- is the Goldstone mode as a consequence of the breakdown of V-spin symmerty; we can choose such form for the condensate as $\langle K^\pm \rangle = f\langle\theta\rangle/\sqrt{2}\cdot\exp(\pm i\gamma)$ with an arbitrary γ. However, once a phase is chosen, the kaon-condensed state is a symmetry-broken phase with respect to the rotation around the third axis in V-spin space by the $U(1)$ operator, $U_V(\nu) = \exp(iV_3\nu)$ with arbitrary ν, while the effective Lagrangian is still inavariant under the transformation, $U \to U_V U U_V^\dagger$. Thus we observe a spontaneous symmetry breaking (SSB) there.

It is easy to show $E_- \sim 0$ for $\mathbf{p} = 0$. In the classical (tree-level) approximation $c_1(\mathbf{p} = 0) = 0$ (see Appendix B), which directly means $E_-(\mathbf{p} = 0) = 0$; actually we can expand E_-,

$$E_-^2 \simeq \frac{c_3}{2C^2}p^2 + \frac{p^4}{4C^2} + ..., \tag{3.24}$$

for small momentum, $|\mathbf{p}| \ll C$. This dispersion relation may remind us of the Bogoliubov spectrum for the phonon mode in superfluidity or Bose-Einstein condensation [30]. When we take into account the fluctuation effects (kaon loops) in the thermodynamic potential, their order of magnitude is $O(\hbar)$. Accordingly, this relation should be modified in $O(\hbar)$, because any *kaon loop* is not included in Eq. (3.23). [11] Hence we should find $c_1(\mathbf{p} = 0) = 0 + O(\hbar)$, which means that once fluctuations are included in the thermodynamic potential, the soft mode loses the Goldstone-boson nature. This situation is inevitable in the perturbative treatment [27]: our thermodynamic potential takes into account the one-loop diagrams, whereas the self-energy diagram for kaons never include any kaon loop. However, we may expect its deviation to be small. Actually, we have found that the zero-point contribution from kaon loops is very small compared with tree-level contributions [13,14]. When we consider the thermal kaon loop, E_- directly enters into the Bose-Einstein distribution function, $f_B(E) = [\exp(\beta E) - 1]^{-1}$, and it should diverge at $\mathbf{p} = 0$. The other is the massive mode, $E_+ \gg 100\text{MeV}$, and we may discard it for temperature we are interested in ($T \leq 100\text{MeV}$). Thus only the thermal K^- loops play an important role due to this property.

Moreover, if we can safely neglect the c_3 term, we get a simple expression for $E_\pm$,

$$E_\pm = \tilde{\omega}_\pm \pm \tilde{\mu}_K = \sqrt{p^2 + C^2} \pm (b + \tilde{\mu}_K). \tag{3.25}$$

There are several reasons to support the pertinence of these formulae, especially for E_-. First, we can see from Eq. (3.24) that in the high momentum limit, $|\mathbf{p}| \gg \sqrt{2c_3} = \mu_K \sin\langle\theta\rangle$, the difference of E_- between Eq. (3.23) and Eq. (3.25) converges to be trifling. [12] On the other hand, in the low momentum limit, $|\mathbf{p}| \ll C$, E_- can be expanded as

$$E_- \simeq \frac{p^2}{2C} + ... \tag{3.26}$$

within the tree approximation, which just corresponds to the second term in Eq. (3.24). Therefore, the approximated formula (3.25) may be well justified for the momentum, $|\mathbf{p}| \gg \mu_K \sin\langle\theta\rangle$, while it spoils the Goldstone-boson behavior ($E_- \propto |\mathbf{p}|$) near $\mathbf{p} = 0$. So we must carefully treat the small-$\mathbf{p}$ region, when we use the approximation. Comparing (3.23) and (3.25) we can expect that the "nonrelativistic" dispersion relation (3.25) holds under the condition, $v^2 \equiv c_3/2C^2 \ll 1$, namely the small velocity limit. Typically $\mu_K \sim O(100\text{MeV})$ and $C \sim O(m_K)$ [5], so that we find $v^2 \sim O(10^{-2}) \ll 1$.

At finite temperature we can find a more meaningful criterion for the pertinence of Eq. (3.25). Consider a typical integral at finite temperature,

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[11] On the other hand, the nucleon loops can be treated consistently (see IV).

[12] This property is very important for the renormalization of kaon loops, where ultraviolet divergences become relevant, see IV.

$$I = \int \frac{d^3p}{(2\pi)^3} f_B(E_-(\mathbf{p})), \tag{3.27}$$

which is the particle number of K^- fluctuations. Using Eq.(3.25) we find

$$I \sim \frac{(CT)^{3/2}}{(2\pi)^{3/2}} \zeta(3/2) \tag{3.28}$$

for $T \leq C$. However, as mentioned above Eq. (3.25) is no longer good for $|\mathbf{p}| < \mu_K \sin\langle\theta\rangle$. We can estimate the integral for this region

$$I_\delta \equiv \int_0^{\mu_K \sin\langle\theta\rangle} \frac{p^2 dp}{2\pi^2} f_B(E_-(\mathbf{p})) \sim \frac{C\mu_K \sin\langle\theta\rangle}{\pi^2 \beta}. \tag{3.29}$$

Therefore, the approximation (3.25) is verified, if $I_\delta/I \ll 1$, which requires the following condition for c_3 or $\mu_K \sin\langle\theta\rangle$ as the function of density and temperature,

$$c_3 \ll \frac{C\pi}{16} T\zeta^2(3/2) \quad \text{or} \quad \mu_K \sin\langle\theta\rangle \ll \frac{1}{4}(2\pi CT)^{1/2}\zeta(3/2). \tag{3.30}$$

We can expect Eq. (3.30) for relevant densities and temperatures [14], except very low temperature, where thermal effects should be trivially unimportant.

Secondly, we can also expect the smallness of c_3 qualitatively: in the limit, $\langle\theta\rangle \to 0$ or $\mu_K \to 0$, c_3 gives no contribution. We also know that $\langle\theta\rangle$ and μ_K are inversely proportional to each other [5]. Hence we might expect that the c_3 term becomes relatively small.

C. Thermodynamic relations

The effective thermodynamic potential $\Omega_{chiral} = -T \ln Z_{chiral}$ reads from Eq. (3.21),

$$\Omega_{chiral} = \Omega_c + \Omega_{sc} + \Omega_K + \Omega_N, \tag{3.31}$$

where Ω_c is the classical kaon contribution,

$$\Omega_c = V[-f^2 m_K^2(\cos\langle\theta\rangle - 1) - 1/2 \cdot \mu_K^2 f^2 \sin^2\langle\theta\rangle], \tag{3.32}$$

Ω_{sc} the subtraction term to avoid the double-counting of the interaction terms within the Hartree approximation,

$$\Omega_{sc} = T\text{Tr}[G_H^{-1}\langle\delta G\rangle] = V\left[\sigma\cos\langle\theta\rangle n_K^1 + bn_K^2\right]. \tag{3.33}$$

The kaon contribution, $\Omega_K = -T \ln Z_K^{eff}$, can be written as,

$$\Omega_K = \Omega_K^{ZP} + \Omega_K^{th}, \tag{3.34}$$

where the zero-point energy (ZP) contribution Ω_K^{ZP} and the thermal one Ω_K^{th} are given as follows;

$$\Omega_K^{ZP} = V \int \frac{d^3p}{(2\pi)^3} \frac{1}{2}[E_+(\mathbf{p}) + E_-(\mathbf{p})] \tag{3.35}$$

and

$$\Omega_K^{th} = TV \int \frac{d^3p}{(2\pi)^3} \ln(1 - e^{-\beta E_+(\mathbf{p})})(1 - e^{-\beta E_-(\mathbf{p})}). \tag{3.36}$$

The thermodynamic potential for nucleons, $\Omega_N = -T \ln Z_N$, can be also written as follows:

$$\Omega_N = -T\text{Tr}\ln G_H = \Omega_N^{ZP} + \Omega_N^{th}, \tag{3.37}$$

where Ω_N^{ZP} denotes the ZP contribution,

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$$\Omega_N^{ZP} = -2V \sum_{n,p} \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{2}(\epsilon_i + \epsilon_{\bar{i}}) \right], \tag{3.38}$$

and Ω_N^{th} the thermal contribution,

$$\Omega_N^{th} = -2TV \sum_{n,p} \int \frac{d^3p}{(2\pi)^3} \left[\ln(1 + e^{-\beta(\epsilon_i - \mu_i)}) + \ln(1 + e^{-\beta(\epsilon_{\bar{i}} + \mu_i)}) \right]. \tag{3.39}$$

Here, $\epsilon_i(\epsilon_{\bar{i}})$, $i = p, n$ are the single particle energies of the nucleons (anti-nucleons),

$$\epsilon_{p,\bar{p}} = \mp \frac{1}{2f^2} n_K^2 \mp \mu_K (1 - \cos\langle\theta\rangle) + E_p^*,$$

$$\epsilon_{n,\bar{n}} = \mp \frac{1}{4f^2} n_K^2 \mp \frac{\mu_K}{2} (1 - \cos\langle\theta\rangle) + E_n^*, \tag{3.40}$$

with the kinetic energy, $E_i^* = \sqrt{p^2 + M_i^{*2}}$.

Using the thermodynamic relations,

$$S_{chiral} = -\frac{\partial \Omega_{chiral}}{\partial T}, \quad Q_i = -\frac{\partial \Omega_{chiral}}{\partial \mu_i}, \quad E_{chiral} = \Omega_{chiral} + TS_{chiral} + \sum_i \mu_i Q_i, \tag{3.41}$$

we can find charge, entropy and internal energy. We easily see that these quantities become too complicated to be tractable if we use the exact dispersion relation in Eq. (3.23) ; e.g. consider the kaonic charge given by way of the relation, $Q_K = -\partial\Omega_{chiral}/\partial\mu_K$, which results in a complicated form. Besides this, the quantities ,n_K^1 or n_K^2, which can be written by the use of the thermal Green's functions, has a complicated form as well (see Appendix B). Hence a useful approximation is desirable. We have already discussed the pertinence of the approximation for the dispersion relation in Eq. (3.25) and we can see that this approximation allows us to write down the thermodynamic quantities in clear forms. For the kaonic charge it immediately gives

$$Q_K/V = \mu_K f^2 \sin^2\langle\theta\rangle + \cos\langle\theta\rangle n_K + (1+x)\sin^2(\langle\theta\rangle/2)\rho_B, \tag{3.42}$$

where n_K is the number density of thermal kaons,

$$n_K = \int \frac{d^3p}{(2\pi)^3} \left[f_B(E_-(\mathbf{p})) - f_B(E_+(\mathbf{p})) \right]. \tag{3.43}$$

The resultant form (3.42) suggests that the charge of kaonic modes is screened by the condensate; in fact, we can see that their effective charge are given by $\pm e_{eff}$ with $e_{eff} = \cos\langle\theta\rangle e$ by the use of the effective Lagrangian (3.15). Accordingly the modified chemical potential $\tilde{\mu}_K = \cos\langle\theta\rangle\mu_K$ gets the meaning of the chemical potential for the *number* of thermally excited kaons.

IV. EFFECTS OF ZERO-POINT FLUCTUATIONS

A. Renormalization

In our formalism the thermodynamic potential (3.31), as well as the quantities n_K^1, σ, is constructed by many one-loop bubbles, which include intrinsic divergences. Hence we must renormalize them to extract finite contributions. Since the building units of these quantities are graphically one-loop bubbles of kaons and nucleons, it suffices to renormalize these diagrams [29]. Then all the diagrams with many loops have no more divergences. Since the temperature dependent part never induces new divergence, we renormalize the temperature independent part. Using the similar method proposed in ref. [13], we can properly renormalize the thermodynamic potential. However, it is to be noted that we must carry out the renormalization program by including not only the kaonic contributions but also the nucleonic one in this case. The counterterms are introduced without spoiling the original symmetry structure of the Lagrangian; the symmetric terms are also invariant under ζ and the symmetry-breaking terms should transform like Eq. (2.23). Since we have already known the stucture of the effective Lagrangian, given in Eqs. (3.4)-(3.6), it is easy to find the proper counterterms. It is also to be noted that the vector-type contributions, $\delta\mathcal{L}$ induced by introducing

the chemical potentials as well as the original Tomozawa-Weinberg term, never suffer from divergences (c.f. Eq. (C13), see also ref. [29]). Thereby it is almost clear the introduction of chemical potentials never generates new divergences. In the following we use the approximated formulae (3.25) for the dispersion relations of kaonic modes for simplicity.

First, we renormalize the self-energy terms of kaons and nucleons given in Eqs. (3.12) and (3.15), respectively. The kaon effective mass is given by

$$m_K^{*2} = m_K^2 - \sigma. \tag{4.1}$$

The self-energy term σ is given by the nucleon loop, $\sigma = f^{-2}\text{Tr}\left[G_H^{-1}\gamma_0\Sigma_{KN}\right] = \sigma_{tree} + \sigma_{loop}$ with the inverse propagator G_H (3.11). We only consider the loop correction σ_{loop} in the following. To the one-loop order we don't care about the n_K^1 that is given by the kaon loop in the effective mass M_i^* (3.12). Then the temperature independent part $\sigma_{loop}|_{T=0}$ reads

$$\sigma_{loop}|_{T=0} = -4\sum_i \Sigma_{Ki}f^{-2}\int \frac{d^3k}{(2\pi)^3}\frac{M_i^*}{2E_i^*}$$

$$= -4\sum_i M_i^*\Sigma_{Ki}f^{-2}\left[d_2 - M_i^{*2}d_0 + \frac{1}{2}M_i^{*2}\ln\left(\frac{M_i^{*2}}{4\kappa^2}\right)\right] \tag{4.2}$$

with the cut-off (Λ) regularization, where d_i's are the quadratically and logarithmically divergent integrals,

$$d_2 = \int_{k\leq\Lambda}\frac{d^3k}{(2\pi)^3}\frac{1}{2k} = \frac{\Lambda^2}{8\pi^2},$$

$$d_0 = \int_{k\leq\Lambda}\frac{d^3k}{(2\pi)^3}\frac{1}{4k^3} = \frac{1}{8\pi^2}\ln\left(\frac{\Lambda}{\kappa}\right), \tag{4.3}$$

with an arbitrary momentum-scale κ. To renormalize the effective mass we redefine the kaon mass and the KK interaction vertex. Then the counterterms should be given as

$$\sigma_{ct} = 4\sum_i M_i^*\Sigma_{Ki}f^{-2}[\alpha_2 - M_i^{*2}\alpha_0]. \tag{4.4}$$

The meaning of the counterterms α_i is graphically clear (Fig. 4): α_2 renormalizes the kaon mass and α_0 the KK vertex at momentum $\mathbf{p} = 0$. It is to be noted that the wavefunction renormalization is not needed to the one-loop order.
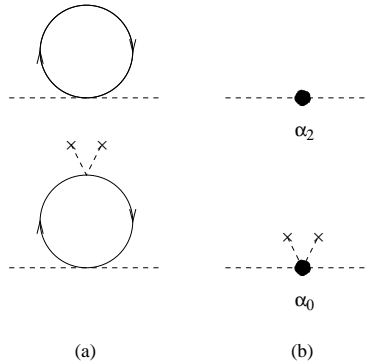

(a)   (b)

FIG. 4. Divergent diagrams (a) and the counterterms (b) to the one-loop order for the kaon propagator (dashed line). Solid line denotes the nucleon propagator without n_K^1 and the cross ($\times$) denotes the condensate $\langle\theta\rangle$. The black blob stands for the contribution from the counterterm.

Then they are determined by the following conditions,

$$(\sigma_{loop}|_{T=0} + \sigma_{ct})|_{vac} = 0, \qquad \frac{\partial^2}{\partial\langle\theta\rangle^2}(\sigma_{loop}|_{T=0} + \sigma_{ct})|_{vac} = 0. \tag{4.5}$$

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From Eqs. (4.2),(4.4)we find

$$\alpha_2 = d_2 - \frac{M^2}{16\pi^2}, \qquad \alpha_0 = d_0 + \frac{1}{8\pi^2}\left[\ln\left(\frac{2\kappa}{M}\right) - \frac{1}{2}\right], \tag{4.6}$$

and consequently the finite contribution is given by

$$\sigma_{loop}^{ren}|_{T=0} = -\sum_i M_i^* \frac{\Sigma_{Ki} f^{-2}}{4\pi^2}\left[M_i^{*2}\ln\left(\frac{M_i^{*2}}{M^2}\right) + M^2 - M_i^{*2}\right]. \tag{4.7}$$

Thus the self-energy term σ is renormalized to be σ_{ren}, $\sigma_{ren} = \sigma_{tree} + \sigma_{loop}^{ren}|_{T=0}$. Note that the dispersion relation is modified by the replacement of σ by σ_{ren}, compared with the tree approximation where σ is given by $\sigma = \sigma_{tree}$ [14]. However, we can see that the Goldstone-boson nature of E_- is never spoiled by such procedure (see Eq. (4.27)).

Next, we renormalize the nucleon effective mass, $M_i^*(i = n, p)$, which is given by Eq. (3.12). It includes the kaon loop correction n_K^1 [13], the temperature independent part of which is given by

$$n_K^1|_{T=0} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{E_+ + E_-}$$
$$= d_2 - C^2 d_0 + \frac{1}{16\pi^2}C^2\ln\left(\frac{C^2}{4\kappa^2}\right), \tag{4.8}$$

(see Eq. (C11)). Accordingly we introduce the counterterms, which renormalize the nucleon mass and the KN interaction vertex at momentum $\mathbf{p} = 0$ (see Fig. 5),

$$M_{ct}^i = f^{-2}\Sigma_{Ki}\cos\langle\theta\rangle[\beta_2 - C^2\beta_0]. \tag{4.9}$$

Here C^2 is given by

$$C^2 = (m_K^2 - \sigma)\cos\langle\theta\rangle + b^2, \tag{4.10}$$

but the nucleon loop in σ is irrelevant up to the one-loop order as well. It is to be noted that the overall factor $\cos\langle\theta\rangle$ in Eq. (4.9) stems from the transformation ζ (see, Eqs. (3.4) and (3.6)).
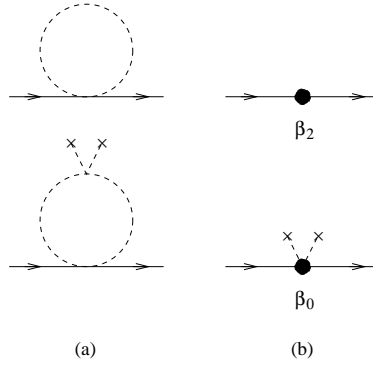

FIG. 5. Divergent diagrams (a) and the counterterms (b) to the one- loop order for the nucleon propagator. Notations are the same as in Fig. 4 .

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[13]Note that another kaon-loop contribution n_K^2 never includes divergence (see Eq. (C13)) because it originates from the vector-type KN interaction, while n_K^1 from the scalar-type interaction.

Then renormalization conditions are given as follows,

$$(M_i^*|_{T=0} + M_{ct}^i)|_{vac} = M, \qquad \frac{\partial^2}{\partial \langle \theta \rangle^2}(M_i^*|_{T=0} + M_{ct}^i)|_{vac} = -\Sigma_{Ki}. \tag{4.11}$$

From Eqs. (3.12),(4.8), (4.9) we find

$$\beta_2 = d_2 - \frac{m_K^2}{16\pi^2}, \qquad \beta_0 = d_0 + \frac{1}{8\pi^2}\left[\ln\left(\frac{2\kappa}{m_K}\right) - \frac{1}{2}\right], \tag{4.12}$$

and thereby the finite contribution to n_K^1 reads

$$n_K^{1,ren}|_{T=0} = \frac{1}{16\pi^2}\left[m_K^2 - C^2 - C^2\ln\left(\frac{m_K^2}{C^2}\right)\right]. \tag{4.13}$$

It is to be noted that the structure of (4.12) is the same as (4.6).

Once σ and n_K^1 are thus renormalized, we can use the renormalized ones given by Eqs. (4.7) and (4.13) in the thermodynamic potential (3.31) instead of the unrenormalized ones. Although the thermodynamic potential (3.31) still includes the divergences through the ZP contributions in Eq. (3.35) and Eq. (3.38), the total ZP contribution $\Omega_{ZP} = \Omega_K^{ZP} + \Omega_N^{ZP}$ should be renormalized by the use of the counterterms α_i and β_i without introduction of another counterterms (see Fig. 6).

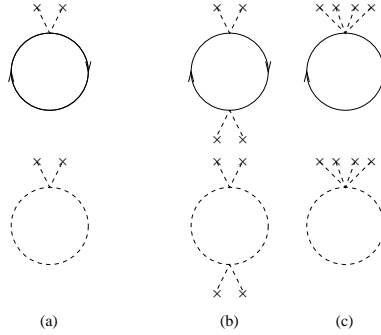
(a)   (b)   (c)

FIG. 6. Divergent diagrams to the one-loop order for the thermodynamic potential. Notations are the same as in Fig. 4. The diagrams in (a) are quadratically divergent and renormalized by the counterterms given in Figs. 4 and 5 , while the diagrams given in (b) and (c), which are renormalized by redefining KK interaction vertex at momentum $\mathbf{p} = 0$, give the quadratic and logarithmic divergences, respectively; the counterterms for (b) and (c) are already given in Figs. 4 and 5 .

Thus we find the renormalized ZP contribution, Ω_{ZP}^{ren},

$$\Omega_{ZP}^{ren} = \Omega_{ZP,K}^{ren} + \Omega_{ZP,N}^{ren}, \tag{4.14}$$

with the kaon-loop contribution,

$$\Omega_{ZP,K}^{ren}/V = \frac{1}{64\pi^2}\left[(C^2 - m_K^2)(m_K^2 - 3C^2) + 2C^4\ln\left(\frac{C^2}{m_K^2}\right)\right], \tag{4.15}$$

and the nucleon-loop one,

$$\Omega_{ZP,N}^{ren}/V = -\sum_{i=n,p}\frac{1}{32\pi^2}\left[(M_i^{*2} - M^2)(M^2 - 3M_i^{*2}) + 2M_i^{*4}\ln\left(\frac{M_i^{*2}}{M^2}\right)\right]. \tag{4.16}$$

Here we have subtracted the vacuum energy which is quartically divergent,

$$\Omega_{vac}/V = -3d_4 - \frac{M^4}{16\pi^2} + \frac{m_K^4}{64\pi^2} \tag{4.17}$$

with the divergent integral,

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$$d_4 = \int_{k \le \Lambda} \frac{d^3k}{(2\pi)^3} k = \frac{\Lambda^4}{8\pi^2}. \tag{4.18}$$

Then we can easily verify the relations,

$$\frac{\partial(\Omega_{ZP,K}^{ren}/V)}{\partial C^2} = n_K^{1,ren}|_{T=0}, \qquad \sum_{i=n,p} f^{-2}\Sigma_{Ki} \frac{\partial(\Omega_{ZP,N}^{ren}/V)}{\partial M_i^*} = \sigma_{loop}^{ren}|_{T=0}. \tag{4.19}$$

It is to be noted that the magnitude of ZP contributions (4.15) and (4.16) are bounded from above,

$$|\Omega_{ZP,K}^{ren}/V| < 12\text{MeV} \cdot \text{fm}^{-3} \quad \text{and} \quad |\Omega_{ZP,N}^{ren}/V| < 620\text{MeV} \cdot \text{fm}^{-3}. \tag{4.20}$$

Comparing the both limit values, we can see that the effect of the zero-point fluctuation by nucleons should be more pronounced than that by kaons.

In the infinite-mass limit for nucleons, $M \to \infty$, there is only left the kaon contribution and Ω_{ZP}^{ren} is reduced to a simple form [14], $\Omega_{ZP}^{ren} \to \Omega_{ZP,K}^{ren}$. It may be worth noting the relations, $\beta_2 - d_2 = f_2$ and $(\beta_0 - d_0) + (\beta_2 - d_2)/m_K^2 = f_0$ in terms of f_i given in ref. [13], which is graphically understood in Fig. 6 .

Finally, we give a remark about the use of the approximated formulae (3.25) for the dispersion relations of the kaonic modes. As has been already mentioned in III, the difference between (3.25) and the exact formulae (3.23) converges to zero for high-momentum limit, so that the structure of the ultraviolet divergences is the same to each other. Therefore, even if we use the exact formulae, we find the same counterterms in Eq. (4.12).

B. Nucleon-loop contribution

We investigate the effects of zero-point fluctuations in neutron-star matter at $T = 0$, since the negative-sea effect is dominant there. At $T = 0$ the thermodynamic potential is reduced into a simple form,

$$\Omega_{chiral}|_{T=0} = \Omega_c + \Omega_{sc}|_{T=0} + \Omega_N|_{T=0} + \Omega_{ZP}^{ren}, \tag{4.21}$$

where $\Omega_{sc}|_{T=0} = V[\sigma_{ren} n_K^{1,ren}|_{T=0} \cos\langle\theta\rangle]$. The nucleon contribution $\Omega_N|_{T=0}$ can be written as

$$\Omega_N|_{T=0} \simeq V \left[3/5\epsilon_F^n(1-x) + 3/5\epsilon_F^p x - \mu_n(1-x) - \mu_p x \right] \rho_B$$
$$- (2b\mu_K + \sigma_{tree})f^2(1 - \cos\langle\theta\rangle) \tag{4.22}$$

with the baryon number density ρ_B, the proton mixing ratio $x = \rho_p/\rho_B$, and Fermi energies for nucleons, $\epsilon_F^n = \left(3\pi^2\rho_B(1-x)\right)^{2/3}/2M$, $\epsilon_F^p = (3\pi^2\rho_B x)^{2/3}/2M$, in the nonrelativistic approximation. Since we have already known that the kaon contribution $\Omega_{ZP,K}^{ren}$ gives only a tiny effect [13,14], we, hereafter, concentrate on the nucleon one $\Omega_{ZP,N}^{ren}$ by simply dropping kaon loops. After the calculation we will confirm it as a consistency check.

It is well-known that the nuclear symmetry energy plays an important role for the ground-state properties of the condensed phase. Hence we take it into account to get a realistic result [14]. Since we have already included the kinetic energy for nucleons, it suffices to consider only the potential energy contribution. Following Prakash et al. [31] we effectively introduce a symmetry energy contribution,

$$\Omega_N^{symm} = V\rho_B(1 - 2x)^2 S^{pot}(u), \tag{4.23}$$

with the relative density, $u = \rho_B/\rho_0$. The coefficient $S^{pot}(u)$ is given as $S^{pot}(u) = (S_0 - (2^{2/3}-1)(3/5)\epsilon_F^0)F(u)$ with the constraint $F(1) = 1$, to reproduce the empirical symmetry energy $S_0 \simeq 30\text{MeV}$, where ϵ_F^0 is the Fermi energy at ρ_0, $\epsilon_F^0 = (3\pi^2\rho_0/2)^{2/3}/2M$, and $F(u)$ is a simulated function, for which we take here $F(u) = u$ for simplicity.

Then the total thermodynamic potential, we consider here, is given by further adding the leptonic one $\Omega_l|_{T=0}$,

$$\Omega_{tot} = \Omega_{chiral}|_{T=0} + \Omega_N^{symm} + \Omega_l|_{T=0}, \tag{4.24}$$

where $\Omega_l|_{T=0}$ can be written in the standard noninteracting form,

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$$\Omega_l|_{T=0} = V\left[-\frac{\mu_K^4}{12\pi^2} + \theta(|\mu_K| - m_\mu)\left\{\frac{m_\mu^4}{8\pi^2}[(2t^2+1)t\sqrt{t^2+1} - \ln(t + \sqrt{t^2+1})] - \frac{m_\mu^3|\mu_K|t^3}{3\pi^2}\right\}\right], \quad (4.25)$$

with $t = \sqrt{\mu_K^2 - m_\mu^2}/m_\mu$. Here the first term is due to electrons and the second term due to muons, and we have used the relation $\mu_l = \mu_K$ for leptons.

The parameters $\langle\theta\rangle$, x and μ_K are determined by the equilibrium conditions and charge neutrality of the ground state [5,9,13]. First, we get

$$f^2 \sin\langle\theta\rangle(m_K^2 - \sigma_{tree} - 2\mu_K b - \mu_K^2 \cos\langle\theta\rangle) + \frac{\partial(\Omega_{ZP,N}^{ren}/V)}{\partial\langle\theta\rangle} = 0 \quad (4.26)$$

from the condition $\partial\Omega_{tot}/\partial\langle\theta\rangle = 0$. Recalling the relation in Eq. (4.19), we find it is further written as

$$f^2 \sin\langle\theta\rangle(m_K^{*2} - 2\mu_K b - \mu_K^2 \cos\langle\theta\rangle) = 0, \quad (4.27)$$

with the effective kaon mass $m_K^{*2} = m_K^2 - \sigma_{ren}$. Then it is to be noted that the critical density is unchanged even if the zero-point fluctuation $\Omega_{ZP,N}^{ren}$ is included, since σ_{ren} is reduced to σ_{tree} as $\langle\theta\rangle \to 0$. It is also to be noted that Eq. (4.27) ensures the Goldstone-boson nature of K^- mode (see Eqs. (3.23), (3.25)), even if the ZP contribution is included. Secondly, we get

$$\epsilon_F^p - \epsilon_F^n - 4S^{pot}(u)(1 - 2x) + \frac{1 - \cos\langle\theta\rangle}{2}\{2(\Sigma_{Kn} - \Sigma_{Kp}) - \mu_K\} + \mu_K = 0. \quad (4.28)$$

from the condition $\partial\Omega_{tot}/\partial x = 0$, which is equivalent with the chemical equilibrium condition among kaons and nucleons, $\mu_n - \mu_p = \mu_K$. Finally, charge neutrality demands the relation,

$$Q_K + Q_l = Q_p(= xV\rho_B), \quad (4.29)$$

which implies $\partial\Omega_{tot}/\partial\mu_K = 0$ by definition, where the lepton charge is given by

$$Q_l/V = \frac{\mu_K^3}{3\pi^2} + \theta(|\mu_K| - m_\mu)\frac{\mu_K}{|\mu_K|}\frac{m_\mu^3 t^3}{3\pi^2}, \quad (4.30)$$

and the kaonic charge,

$$Q_K/V = \mu_K f^2 \sin^2\langle\theta\rangle + (1 + x)\sin^2(\langle\theta\rangle/2)\rho_B. \quad (4.31)$$

Then the energy is given by way of the thermodynamic relation,

$$E_{tot} = \Omega_{tot} + \sum_{n,p,K,l} \mu_i Q_i. \quad (4.32)$$

Note that these equations are the same as the previous ones in the heavy-nucleon limit [14] if we simply put $\Omega_{ZP,N}^{ren} = 0$.

In Table I and II we present the optimum values of μ_K, x, and $\langle\theta\rangle$ for given densities, in comparison with the classical values. We use the values, $a_1 m_s = -67$ MeV, $a_2 m_s = 134$ MeV and $a_3 m_s = -134(-222)$ MeV in Table I(II). [14]

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[14]Recent lattice simulations suggest larger values for KN sigma term [32].

TABLE I. Optimum values of the parameters for the tree case and the one-loop case. We use the value, $a_3 m_s = -134$MeV.

	tree			tree + loop		
u	$\langle\theta\rangle$[deg.]	μ_K [MeV]	x	$\langle\theta\rangle$[deg.]	μ_K [MeV]	x
4.19	0.00	255.8	0.193	0.00	255.8	0.193
4.40	18.85	244.0	0.231	18.74	244.2	0.231
4.80	31.97	221.1	0.293	31.49	222.6	0.290
5.20	40.85	197.7	0.343	39.91	201.6	0.338
5.60	47.62	174.4	0.384	46.26	181.3	0.376
6.00	52.92	151.9	0.417	51.26	161.9	0.407
6.40	57.09	130.6	0.442	55.25	143.5	0.431
6.80	60.36	110.8	0.462	58.46	126.0	0.450
7.20	62.91	92.6	0.477	61.05	109.7	0.465

TABLE II. Optimum values of the parameters for $a_3 m_s = -222$MeV.

	tree			tree + loop		
u	$\langle\theta\rangle$[deg.]	μ_K[MeV]	x	$\langle\theta\rangle$[deg.]	μ_K [MeV]	x
3.08	0.00	218.8	0.156	0.00	218.8	0.156
3.20	18.28	206.4	0.198	17.42	207.9	0.195
3.60	39.75	160.0	0.326	33.60	181.3	0.286
4.00	53.71	109.2	0.425	42.20	161.8	0.344
4.40	63.74	59.1	0.494	48.33	144.7	0.386
4.80	71.05	14.4	0.538	53.12	128.5	0.419
5.20	76.13	-23.3	0.565	57.04	112.3	0.444
5.60	79.67	-54.9	0.581	60.30	95.8	0.465
6.00	82.19	-81.5	0.591	63.08	79.2	0.482

Then we can observe a sizable effect of the zero-point fluctuation by nucleons; it shifts the optimum values of x and $\langle\theta\rangle$ downward, while μ_K upward, comapared with the classical values. The effect becomes prominent as the value of the KN sigma term is increased, since the reduction rate of the nucleon effective mass becomes high. As mentioned above it never affects the critical density, while it suppresses the growth of the condensate.

In Fig.7 the energy difference between the condensed phase and normal one is dipicted as a function of density. We can see that the energy gain is also reduced along with the suppression of the condensate; the zero-point fluctuation of nucleons reduces the energy gain by shifting the optimum values of parameters, while the resultant ZP contribution is small in magnitude. We also estimate the magnitude of the kaon loops by the use of the values given in Table I, II to be tiny for a consistency check.
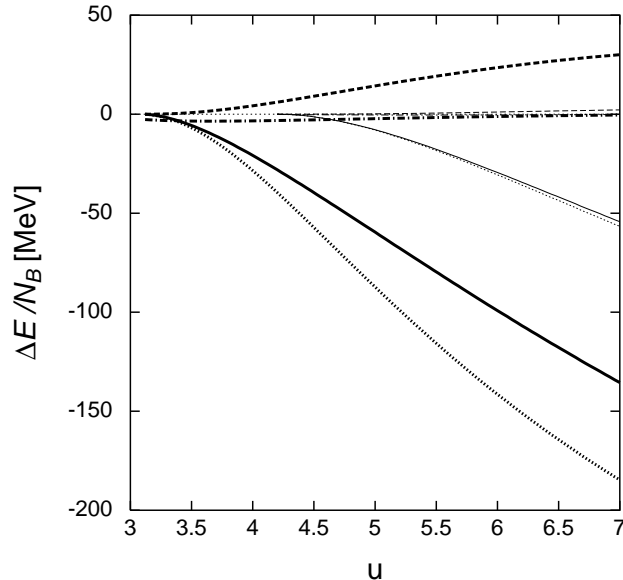

FIG. 7. Energy differences per baryon, $\Delta E/N_B \equiv (E_{tot} - E_{tot}(\langle\theta\rangle = 0))/N_B$, and ZP contributions for $a_3 m_s = -134\text{MeV}$ (thin lines) and $a_3 m_s = -222\text{MeV}$ (thick lines). Solid lines show the results with the zero-point fluctuation by nucleon, while dotted lines those in the tree level. Dashed lines show the ZP contribution. Kaon ZP contributions are estimated by the use of optimum values given in Table I, II (dash-dotted lines).

We notice that these consequences originate from the replacement of the KN interaction term σ_{tree} in the tree approximation by σ_{ren} in the effective mass of kaons m_K^*; the zero-point fluctuation works to reduce the attractive effect. It may be worth noting that m_K^{*2} stays to be in positive value and μ_K also becomes positive, once the zero-point fluctuation is included. It may remind us of the relativistic effect by nucleons [33], which also gives rise to the same effect. Hence, if we further take into account the relativistic effect in our formalism, it may be milder than before. Dispesion relations are also checked to be fairly good by comparing the results given by Eqs. (3.23) and (3.25).

V. SUMMARY AND CONCLUDING REMARKS

In this paper we have presented a formalism to treat fluctuations around the condensate within the framework of chiral symmetry. Our approach is based on the group theoretical argument; Goldstone fields are regarded as coordinates on the chiral $SU(3)_L \times SU(3)_R/SU(3)_V$ manifold and each state corresponding to the condensed phase or the normal phase with fluctuations can be represented as a chiral rotated state from the vacuum, which corresponds to the fiducial point on the manifold. Hence the fluctuations in the vicinity of the condensate are introduced by way of the successive chiral transformations by ζ and η. This procedure can be regarded as the introduction of the local coordinates around the condensed point on the chiral manifold. Since the chiral transformation is the isometry, the geometry around the condensed point

is the same as in the vicinity of the vacuum, where we might use the flat curvature. Hence this method has the advantage of avoiding the Lee-Yang term.

More interestingly, this method corresponds to the separation of the zero mode; since there should appear the Goldstone mode as a result of the symmetry breaking of V-spin beyond the critical density, we must treat the zero modes nonperturbatively even in the normal phase. In the infinite-volume limit the condensate only contributes to the thermodynamical potential among zero modes in the condensed phase. Therefore we have put U into the form given in Eq. (2.14) from the beginning. It is worth mentioning that this method might have a wider applicability for studying phase transitions, e.g. in finite volume, if system has some symmetry.

We have considered the thermodynamic potential up to the one-loop diagrams of kaons and nucleons and the KN interactions have been treated self-consistently within the Hartree approximation; consequently it consists of infinite series of many one-loops. Then the dispersion relation for kaon excitation, which is a fundamental object to examine the thermal properties of the condensed phase, can be obtained. There appear two modes: Goldstone-like soft mode and very massive mode, which correspond to K^-- and K^+-mesonic excitations, respectively, in the condensed phase. Hence, the thermal loops due to the soft mode play an important role. We have seen that the form of the dispersion relation can be reduced to a simple one, if we can take an approximation, $c_3 = 0$. It should be worth noting that once this approximation works, the expressions of other quantities like charge, entropy, internal energy become very simple.

The thermodynamic potential as well as the self-energy terms for kaons and nucleons contains some divergences. We have shown that these divergences are properly renormalized by way of the cut-off regularization; counterterms are introduced to redefine the masses of kaons and nucleons and the KN and KK interaction vertices. Once the self-energy terms are renormalized, we need not require more counterterms to renormalize the thermodynamic potential.

We have discussed the effect of the zero-point fluctuation by nucleons. It provides a sizable effect for the optimum values of the parameters, so that the energy gain is reduced from the classical values. This is because the KN sigma term remarkably reduces the nucleon effective mass in the condensed phase, while the critical density is not affected. On the other hand the contribution by the kaon loops is still tiny.

It may be interesting to study thermal effects in the condensed phase, which is closely related with the problem of protoneutron stars, especially the effects of thermal loops of $n_K^{1,ren}, n_K^2$ [34]. In a subsequent paper [18] we discuss the thermal properties of the kaon condensed phase on the basis of the formalism given in this paper. We shall see the phase diagram in the density-temperature plane or equation of state for the kaon-condensed phase.

APPENDIX A: SEPARATION OF ZERO MODES

Here we reconstruct our formalism from the viewpoint of separation of the zero mode, following ref. [19]. To make our argument clear we first consider the finite volume $V = L^3$ and put it infinity at the end. Since we are interested in the kaon condensation, we restrict ourselves to the kaon and nucleon degrees of freedom for simplicity. Considering the partition function (2.9) and integrating out the baryonic degrees of freedom, we get the partition function for U,

$$Z_{chiral} = N' \int [dU] \exp[S^{eff}(U)], \tag{A1}$$

with the effective action $S^{eff}(U) = S(U) + \text{Tr} \ln G(U)$, where $S(U)$ stands for the pure mesonic action and the second term stems from the integration over the baryon field. The capital letter for trace is used for the Dirac operator $G(U)$ of infinite dimension over space-imaginary-time. Substituting $U = \exp(2i\phi/f)$ with $\phi = T_a \phi_a$ in Eq. (A1), the action has the following structure,

$$S^{eff}(U) = \bar{S}_K(K) + \text{Tr} \ln \bar{G}, \tag{A2}$$

with

$$\bar{S}_K = \int_0^\beta d\tau \int d^3x \left[-\partial_\mu K^+ \partial_\mu K^- - (m_K^2 - \mu_K^2)K^+ K^- - \mu_K K^+ \overset{\leftrightarrow}{\partial_\tau} K^- + O(|K|^4) \right]. \tag{A3}$$

The matrix $\bar{G}$ reads

$$\bar{G} = \bar{G}_0 + \delta\bar{G} \tag{A4}$$

with the free part

$$\bar{G}_0 = -\frac{\partial}{\partial \tau} + i\gamma_0 \boldsymbol{\gamma} \cdot \boldsymbol{\nabla} - \gamma_0 M + \mu_N \tag{A5}$$

and the interaction part

$$\delta\bar{G} = \frac{\Sigma_{KN}}{f^2} \gamma_0 K^+ K^- - \frac{1}{f^2} \frac{3 + \tau_3}{8} \left(K^+ \overset{\leftrightarrow}{\partial_\tau} K^- - 2\mu_K K^+ K^- \right) + O(|K|^4). \tag{A6}$$

For the purpose of the Hartree approximation, we define $\bar{G}_H$ as

$$\bar{G}_H = \bar{G}_0 + \langle \delta\bar{G} \rangle, \tag{A7}$$

by taking the thermal average for the kaon field. Then the effective action of the nucleonic part can be written as

$$\text{Tr} \ln \bar{G} = \text{Tr} \ln \bar{G}_H + \text{Tr}[\bar{G}_H^{-1} \delta G] - \text{Tr}[\bar{G}_H^{-1} \langle \delta\bar{G} \rangle] + O(|K|^4). \tag{A8}$$

Thus effective action S^{eff} can be written as

$$S^{eff} = \bar{S}_K + \text{Tr} \ln \bar{G}_H + \text{Tr}[\bar{G}_H^{-1} \delta G] - \text{Tr}[\bar{G}_H^{-1} \langle \delta G \rangle] + O(|K|^4). \tag{A9}$$

Substituting Eqs. (A6) into Eq. (A9), we find

$$S^{eff} = \text{Tr} \ln \bar{G}_H - \text{Tr}[\bar{G}_H^{-1} \langle \delta G \rangle]$$
$$+ \int_0^\beta d\tau \int d^3x \left[-\partial_\mu K^+ \partial_\mu K^- - M_K^2 K^+ K^- - (\bar{b} + \mu_K)K^+ \overset{\leftrightarrow}{\partial_\tau} K^- + O(|K|^4) \right], \tag{A10}$$

where M_K^2 is the "mass" term for the kaon field, $M_K^2 \equiv m_K^2 - \Pi_{KN}$, with the self-energy term, $\Pi_{KN} = -\bar{\sigma} - 2\mu_K\bar{b}$. Here $\bar{\sigma}$ and $\bar{b}$ stand for $\bar{\sigma} = f^{-2}\text{Tr}[\gamma_0 \Sigma_{KN} \bar{G}_H^{-1}]$ and $\bar{b} = f^{-2}\text{Tr}[(3 + \tau_3)/8\bar{G}_H^{-1}]$, respectively. It is to be noted that the "mass" term should include the contributions from the meson-baryon interactions (Π_{KN}) and the chemical potential besides the genuine mass of kaons, m_K^2.

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To perform the integral in Eq. (A1) we expand the Goldstone field in terms of periodic plane waves like Eq. (3.16),

$$\phi_a = \sqrt{\frac{\beta}{V}} \sum_n \exp[2\pi i(\mathbf{n} \cdot \mathbf{r}/L + n_4\tau T)]\phi_a^n. \tag{A11}$$

Inserting Eq. (A11) into Eq. (A10), we can see that the magnitude of ϕ_a^n is at most $O(T/\sqrt{M_K^2 + L^{-2}})$. Hence if $T \ll M_K$, the Gaussian integral should give the dominant contribution to the partition function.

However, if the phase transition occurs and the "mass" term M_K^2 tends to be vanished at the critical point, the standard chiral perturbation fails, because there is no quadratic term for the zero-mode ($n_\mu = 0$); in fact, the "mass" term takes negative values before the phase transition, while it is vanished at the critical density. Hence we need to reorder the perturbation series by treating the zero mode nonperturbatively to get a consistent formulation valid even at the critical point.

Following Gasser and Leutwyler we treat the zero-modes as the collective variables [19],

$$U = vU_f v \tag{A12}$$

with $U_f = \exp[2i(T_4\eta_4 + T_5\eta_5)/f]$ and the $U(1)$ matrix $v = \exp(i\theta_0 T_4)$. v is the *constant* unitary matrix representing the zero mode and η_a sums up the nonzero modes,

$$\eta_a = \sqrt{\frac{\beta}{V}} \sum_{n\neq 0} \exp[2\pi i(\mathbf{n} \cdot \mathbf{r}/L + n_4\tau T)]\phi_a^n. \tag{A13}$$

After integrating out with respect to the nonzero modes, we find the integral over the zero mode ($U(1) \subset SU(3)$) [15]. Substituting Eq. (A12) into Eq. (A2), we can see that the form of the effective action is the same with Eq. (3.7) with θ_0 instead of $\langle\theta\rangle$. Hence the integral over the zero mode becomes

$$Z_{chiral} = \int d\theta_0 \exp[-\bar{\Omega}_{chiral}(\theta_0)/T], \tag{A14}$$

where $\bar{\Omega}_{chiral}(\theta_0)$ is given by Eq. (3.31) with replacing the thermal average $\langle\theta\rangle$ by θ_0. Separating the trivial volume dependence from the thermodynamic potentials, we write Eq. (A14) as

$$Z_{chiral} = \int d\theta_0 \exp[-V\beta\bar{\omega}], \tag{A15}$$

with $\bar{\omega} \equiv \bar{\Omega}_{chiral}(\theta_0)/V$. In the large volume limit or the low temperature limit we can apply the steepest descent method to evaluate the integral in Eq. (A15):

$$Z_{chiral} \simeq \exp[-V\beta\bar{\omega}(\theta_0^m)]\sqrt{\frac{2\pi}{V\beta\bar{\omega}''}}, \tag{A16}$$

where ω'' stands for $\partial^2\bar{\omega}/\partial\theta_0^2|_{\theta_0=\theta_0^m}$ and the saddle point θ_0^m is determined by the equation,

$$\partial\bar{\omega}(\theta_0)/\partial\theta_0|_{\theta_0=\theta_0^m} = 0. \tag{A17}$$

This is nothing but an extremum condition for the thermodynamic potential with respect to the order parameter. Hence θ_0^m is equivalent with the thermal average of θ, $\theta_0^m \equiv \langle\theta\rangle$. Accordingly the zero-mode matrix v_m is reduced to ζ. In conclusion, when some degrees of freedom should condense, it is enough to separate them from the beginning in the form Eq. (A12) in the limit, $V \to \infty$.

Here it is also interesting to observe that there is a logarithmic singularity in the thermodynamic potential at the critical point as long as $V \neq \infty$.

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[15]In the general case the zero modes cover the $SU(3)$ manifold, and the partition function is given by the group integral over $SU(3)$ [20].

APPENDIX B: CLASSICAL APPROXIMATION – (MODEL-INDEPENDENT)

In the classical approximation, $\phi_a = 0$, the operator u becomes trivial,

$$U_f = u = 1. \tag{B1}$$

Furthermore, replacing the bilinear operator of the nucleon field by its expectation value, e.g., $\bar{\psi}\psi \to \langle\bar{\psi}\psi\rangle$, then we get

$$\delta\mathcal{L} = \frac{\mu_K}{2}\text{tr}\{B^\dagger[(\zeta^\dagger[T_{em}, \zeta] + \zeta[T_{em}, \zeta^\dagger]), B]\} + \frac{f^2\mu_K^2}{4}\text{tr}\{[T_{em}, \zeta^2][T_{em}, \zeta^{\dagger 2}]\}$$
$$= \mu_K(\cos\langle\theta\rangle - 1)(1 + x)\rho_B - 1/2 \cdot \mu_K^2 f^2 \sin^2\langle\theta\rangle, \tag{B2}$$

which stems from the kinetic terms of Goldstone bosons, and thereby model-independent [9].

The symmetry breaking part is also transformed,

$$\mathcal{L}_{SB}(U_K = \zeta^2, B) = \mathcal{L}_{SB}(1, B) + (\cos\langle\theta\rangle - 1)\langle\Sigma_K(1, B)\rangle$$
$$= (\cos\langle\theta\rangle - 1)(-f^2 m_K^2 + \Sigma_{Kp}\rho_p + \Sigma_{Kn}\rho_n) + \mathcal{L}_{SB}(1, B), \tag{B3}$$

according to the octet dominance hypothesis (see, Eq. (2.28)), where we have used the nonrelativistic approximation for nucleons. Therefore the result is also model independent once we employ this hypothesis [9]. Thus we can recover the previous result: for the energy difference between kaon condensed phase and normal one,

$$\delta E/V = E_{cl}/V + \mu_K Q_{cl}^K/V, \tag{B4}$$

where the "classical" energy E_{cl}, which is nothing but the classical thermodynamic potential, is given by

$$E_{cl}/V = (\cos\langle\theta\rangle - 1)[\mu_K(1 + x)\rho_B/2 - f^2 m_K^2 + \Sigma_{Kp}\rho_p + \Sigma_{Kn}\rho_n)] - 1/2 \cdot \mu_K^2 f^2 \sin^2\langle\theta\rangle, \tag{B5}$$

for $T = 0$. The charge of the condensed kaons, Q_{cl}^K is given by

$$Q_{cl}^K/V = \mu_K^2 f^2 \sin^2\langle\theta\rangle + \mu_K(1 + x)\rho_B(1 - \cos\langle\theta\rangle)/2. \tag{B6}$$

It is to be noted that this result has been obtained without recourse to the definite Lagrangian.

The optimum values of the parameters, $\langle\theta\rangle, x, \mu_K$, are determined by the extremum conditions for E_{cl},

$$\partial E_{cl}/\partial\langle\theta\rangle = 0, \quad \partial E_{cl}/\partial x = 0, \quad \partial E_{cl}/\partial\mu_K = 0. \tag{B7}$$

In particular, we find the field equation for the condensate from the first equation,

$$f^2 \sin\langle\theta\rangle(m_K^{*2} - 2\mu_K b - \mu_K^2 \cos\langle\theta\rangle) = 0 \tag{B8}$$

with $m_K^{*2} = m_K^2 - \sigma_{tree}$, which implies $c_1(\mathbf{p} = 0) = 0$ for $\langle\theta\rangle \neq 0$.

APPENDIX C: KAON GREEN'S FUNCTIONS

The Green's functions of kaons can be extracted from the matrix D^{eff}. Define the inverse- propagator matrix $\mathcal{D}^{-1} \equiv \beta^{-2} D^{eff}$: $\mathcal{D}_{11}^{-1} = \omega_n^2 + c_1 + 2c_3$, $\mathcal{D}_{22}^{-1} = \omega_n^2 + c_1$, and $\mathcal{D}_{12}^{-1} = -\mathcal{D}_{21}^{-1} = -2c_2\omega_n$. Then the propagators are given by

$$\langle \phi_4 \phi_4 \rangle = \mathcal{D}_{11} = \mathcal{D}_{22}^{-1} |\mathcal{D}^{-1}|^{-1} \tag{C1}$$

and

$$\langle \phi_5 \phi_5 \rangle = \mathcal{D}_{22} = \mathcal{D}_{11}^{-1} |\mathcal{D}^{-1}|^{-1}, \tag{C2}$$

with $det(\mathcal{D}^{-1}) \equiv |\mathcal{D}^{-1}|$. Recalling the Dzyaloshinskii relation [23],

$$\mathcal{D}(\omega_n) = D^R(i\omega_n), \tag{C3}$$

with the retarded (real-time) Green's function D^R, we find the excitation energy of the kaon modes are given by $D_{ij}^{R-1} = 0$ or $det(\mathcal{D}^{-1}) = 0$.

Similarly

$$\langle \phi_4 \phi_5 \rangle = \mathcal{D}_{12} = -\mathcal{D}_{21}^{-1} |\mathcal{D}^{-1}|^{-1}, \quad \text{and} \quad \langle \phi_5 \phi_4 \rangle = \mathcal{D}_{21} = -\mathcal{D}_{12}^{-1} |\mathcal{D}^{-1}|^{-1}. \tag{C4}$$

1. Calculation of n_K^0, n_K^1, n_K^2

Consider the following quantities; $n_K^1 \equiv \langle K^+ K^- \rangle$, $n_K^0 \equiv \langle (K^+ \dot{K}^- - \dot{K}^+ K^-) \rangle$ and $n_K^2 = n_K^0 + 2\tilde{\mu} n_K^1$. Hence it is sufficient to evaluate n_K^0 and n_K^1. By way of the kaon Green's functions they are represented as

$$n_K^1 = \langle K^+ K^- \rangle = 1/2\beta^{-1} \sum_n \int \frac{d^3 p}{(2\pi)^3} (\mathcal{D}_{11} + \mathcal{D}_{22})$$

$$= 1/2\beta^{-1} \sum_n \int \frac{d^3 p}{(2\pi)^3} [|\mathcal{D}^{-1}|^{-1} (\mathcal{D}_{11}^{-1} + \mathcal{D}_{22}^{-1})], \tag{C5}$$

and

$$n_K^0 = \langle (K^+ \dot{K}^- - \dot{K}^+ K^-) \rangle = \beta^{-1} \sum_n \int \frac{d^3 p}{(2\pi)^3} 2\omega_n |\mathcal{D}^{-1}|^{-1} \mathcal{D}_{12}^{-1}. \tag{C6}$$

Generally it is rather complicated to evaluate these quantities, but we shall see that we can easily do them under the approximation $c_3 = 0$. In this case, $\mathcal{D}_{11}^{-1} = \mathcal{D}_{22}^{-1}$, thereby the determinant $|\mathcal{D}^{-1}|$ is factorized into $(\mathcal{D}_{11}^{-1} + i\mathcal{D}_{12}^{-1})(\mathcal{D}_{11}^{-1} - i\mathcal{D}_{12}^{-1})$. Hence we find

$$det(\mathcal{D}^{-1}) \rightarrow \left(\frac{1}{\mathcal{D}_{11}^{-1} + i\mathcal{D}_{12}^{-1}} + \frac{1}{\mathcal{D}_{11}^{-1} - i\mathcal{D}_{12}^{-1}} \right) (2\mathcal{D}_{11}^{-1})^{-1}$$

$$= \left(\frac{1}{\mathcal{D}_{11}^{-1} + i\mathcal{D}_{12}^{-1}} - \frac{1}{\mathcal{D}_{11}^{-1} - i\mathcal{D}_{12}^{-1}} \right) (-2i\mathcal{D}_{12}^{-1})^{-1}. \tag{C7}$$

Note that $\mathcal{D}_{11}^{-1} \pm i\mathcal{D}_{12}^{-1} = (\omega_n \pm iE_-)(\omega_n \mp iE_+)$, then n_K^0, n_K^1 can be written as

$$n_K^1 = \beta^{-1} \sum_n \int \frac{d^3 p}{(2\pi)^3} \left[\frac{1}{E_+ + E_-} \left(\frac{E_-}{\omega_n^2 + E_-^2} + \frac{E_+}{\omega_n^2 + E_+^2} \right) \right], \tag{C8}$$

and

$$n_K^0 = \beta^{-1} \sum_n \int \frac{d^3 p}{(2\pi)^3} \left[\frac{4\omega_n^2}{E_+ + E_-} \left(\frac{1}{\omega_n^2 + E_-^2} - \frac{1}{\omega_n^2 + E_+^2} \right) \right]. \tag{C9}$$

Using the relation

$$\sum_{n=-\infty}^{\infty} \frac{1}{x^2 + n^2} = \frac{\pi}{x} \coth \pi x, \tag{C10}$$

we finally get

$$n_K^1 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_+ + E_-} \left[1 + \frac{1}{e^{\beta E_-} - 1} + \frac{1}{e^{\beta E_+} - 1} \right], \tag{C11}$$

and

$$n_K^0 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{(E_+ + E_-)/2} \left[-(E_- - E_+)/2 - \frac{E_+}{e^{\beta E_+} - 1} + \frac{E_-}{e^{\beta E_-} - 1} \right]. \tag{C12}$$

The first terms in Eqs. (C11) and (C12) mean the contributions from the zero-point fluctuation to be renormalized. Finally $n_K^2 = n_K^0 + 2\tilde{\mu}_K n_K^1$ can be written as

$$n_K^2 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{(E_+ + E_-)/2} \left[-\frac{E_+ - \tilde{\mu}_K}{e^{\beta E_+} - 1} + \frac{E_- + \tilde{\mu}_K}{e^{\beta E_-} - 1} \right]. \tag{C13}$$

It is to be noted that there is no contribution by the zero-point fluctuation in Eq. (C13) due to the vector nature of the KN interaction.

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